What Makes Mathematical Truths Synthetic and *A Priori*

If one were to ask a group of people what they consider to be an objective, self-contained truth, mathematical propositions would perhaps be among the first answers that they get. Contrary to such a commonly held belief, I argue that mathematical truths are synthetic, *a priori* judgments, as Kant also suggests in the introduction to the second edition of his *Critique of Pure Reason*. In other words, they are the fruit of synthesis, rather than analysis, and thus not quite self-contained, while indeed being independent of sensual cognition. In support of this, I will explain in detail why mathematical truths are *a priori*, followed by why they are synthetic. Some explanation will also follow as to how Kant argues synthetic, *a priori* judgments are possible at all, as well as why the possibility of such judgments is significant in his arguments raised in the *Critique*.

First, however, I must clarify the meanings of key adjectives used to describe judgments, such as analytic, synthetic, *a priori* and *a posteriori*, as Kant uses them. One would classify a judgment as analytic if its subject either contains or excludes its predicate entirely, while a judgment would be synthetic if otherwise (A6-7/B10). That is to say, analytic judgments would result solely from the definitions of terms, whereas one would have external information, such as sensory experience, connect the subject and the predicate in order to draw synthetic judgments. As for *a priori* and *a posteriori* judgments, “*a priori*” refers to cognitions that do not rely on sensory experience to be justified, whereas “*a posteriori*” refers to those that do (B2). In other words, one must determine the truth value of an *a posteriori* judgment based on empirical data or experience, whereas reason alone would suffice in proving or disproving an *a priori* one.
The terms of each of these pairs are mutually exclusive, meaning that a judgment must be either analytic or synthetic, but it cannot be both or neither, and the same applies to \textit{a priori} and \textit{a posteriori}. Therefore, my initial statement could read as follows: all mathematical truths come to be true by synthesis, yet they remain independent of sensory experience for justification.

All mathematical judgments, Kant argues, are \textit{a priori} (B14-15). This is rather obvious, as mathematical truths are grounded on the capacity of the human mind to reason rather than an empirical source for justification. For example, seeing figures, adding numbers or measuring angles does not make a proposition any truer. At their best, such sensory triggers may help one’s intuition make sense of how it comes to be valid, as it is the intuition that justifies the proposition. In other words, experience is useless in validating the necessity that all mathematical judgments entail. Ultimately it is up to reason to justify the propositions.

Slightly more difficult to understand or defend is Kant’s argument that mathematical judgments are synthetic. This may initially sound contradictory to his argument that those judgments are \textit{a priori}. However, his account for the possibility of synthetic \textit{a priori} judgments sets him apart from his predecessors Leibniz and Hume and still remains a popular topic for philosophical discussions some two centuries later.

To be precise, Kant argues that all truths in \textit{pure} mathematics are synthetic, with the exception of those that have to do with the definitions of concepts; such truths are analytic, justified by the unpacking of ideas. Examples of the areas of mathematics that fall into pure mathematics include arithmetic and pure geometry. While one may
initially consider these truths to be analytic and necessary, and the denial of those propositions to result in contradiction, they in fact come to be true through human intuition, rather than through logic and mere extraction of concepts. Kant uses as an example a simple arithmetic proposition, “7+5=12” (B15). It is obvious that the truth value of this statement obviously relies on the validity of the equal sign, but the concept of the sum of 7 and 5 does not include in itself the number 12. Sebastian Gardner, a Kant scholar, explains this in the following way: “It [the concept of the sum of 7 of 5] does contain of a number which is the union of 7 and 5, but it does not tell us which number that is …” (56, emphasis in the original). One would need the aid of the intuition in order to realize the connection between the sum of the two numbers and the number that it equals, and such use of the intuition to “make the connection” (Ibid., 57) between the subject and the predicate constitutes a synthesis, as mathematical concepts do not contain any concept to which it is equal or synonymous. Therefore, any algebraic or geometric statement that contains connections, such as equation or inequality, is synthetic.

At this point, one may argue that it is contradictory to assume the possibility of a synthetic a priori statement, the understanding of which would require more than mere logic and definitions of concepts, yet justified without sensory experience. In fact, Gottfried Leibniz and David Hume, two leading philosophers preceding Kant, classified all judgments into two categories: products of reason, which were analytic and a priori, and matters of fact, synthetic and a posteriori (Gardner 55). They suggested that the necessity of synthesis with additional data entails the reliance of synthetic
judgments on experience. However, Kant argues, the synthesized term or concept did not have to come from an empirical source. Rather, he believed that one could justify a judgment by looking at other facts that are obviously true, whose truth depended upon the judgment in question being true (B17). Such strong working assumptions are justified not entirely because one could see or feel them to be true, but rather because our common, coherent knowledge about the universe depends on their truth. For example, one would justify the concept of gravity, the force that pulls an object toward another, through a pattern of events observed daily, the explanation of which requires the idea of the invisible, intangible force to be true. Likewise, a judgment may be justified through obvious facts that rely upon the truth of the judgment, where the reason provides the external piece of information needed for synthesis.

Why, then, does Kant raise such a revolutionary argument in the introduction of the text? Whereas this argument may appear to be Kant’s attempt to establish a mathematical framework, it serves a purpose beyond the apparent one. Its ultimate purpose, Gardner explains, is rather to prove the possibility of synthetic a priori judgments for the sake of arguments he raises in the later parts of the Critique, where he maintains that metaphysical propositions are also synthetic and a priori (60). In other words, one must take Kant’s claim regarding the synthetic apriority of mathematical judgments as groundwork for the skeleton of his arguments in the Critique, rather than a mathematical argument in itself. In fact, Kant takes an analytical approach to mathematics and geometry in his later work, the Prolegomena (Ibid., 59). In this regard, the Kantian point of view on mathematics, which recent developments in natural
science and mathematics have challenged and questioned, is not central to Kant’s framework. His arguments raised in the rest of the *Critique* remain intact, as he has successfully proven the possibility of synthetic *a priori* judgments.

In conclusion, truths in pure mathematics, such as ones in arithmetic and geometry, are synthetic and *a priori*, as Kant suggests. While the argument may initially seem contradictory, an argument can, in fact, be both synthetic and *a priori*; moreover, one must keep in mind that this claim is not a mathematical argument in its nature, but rather groundwork for his perspective on metaphysical judgments.


*Note.* The standard A/B [1781/1787] pagination is used for all references to the *Critique* in the essay.